Mittlinjeprogram för förlustmodelering

Lund University

The project

Two years:

WP1	Literature survey
WP 2	Shock loss model
WP 3	Secondary loss model
WP 4	Implementation

Involved people:

LTH: Luis Teia, Magnus Genrup and Marcus Thern

GKN: Pieter Groth

Acknowledgements to the funding bodies:

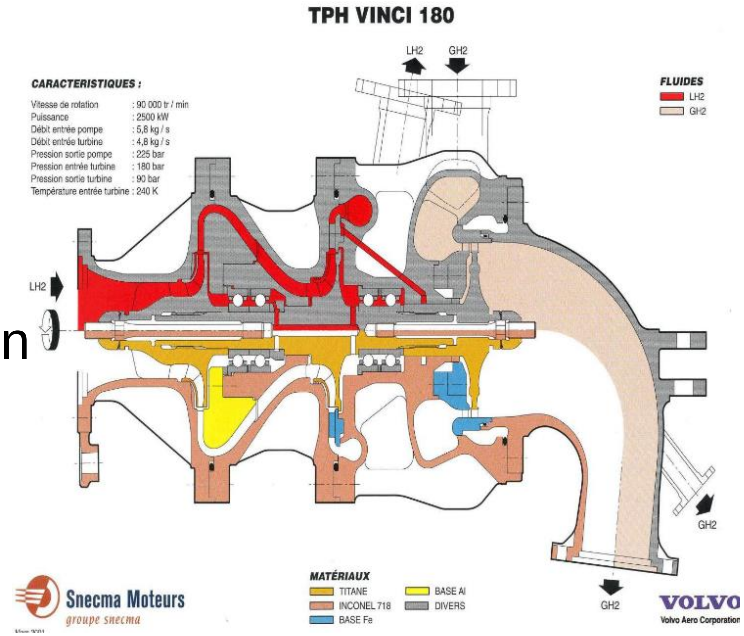
- Rymdstyrelsen (NRFP3)
- LTH

Background and project aim

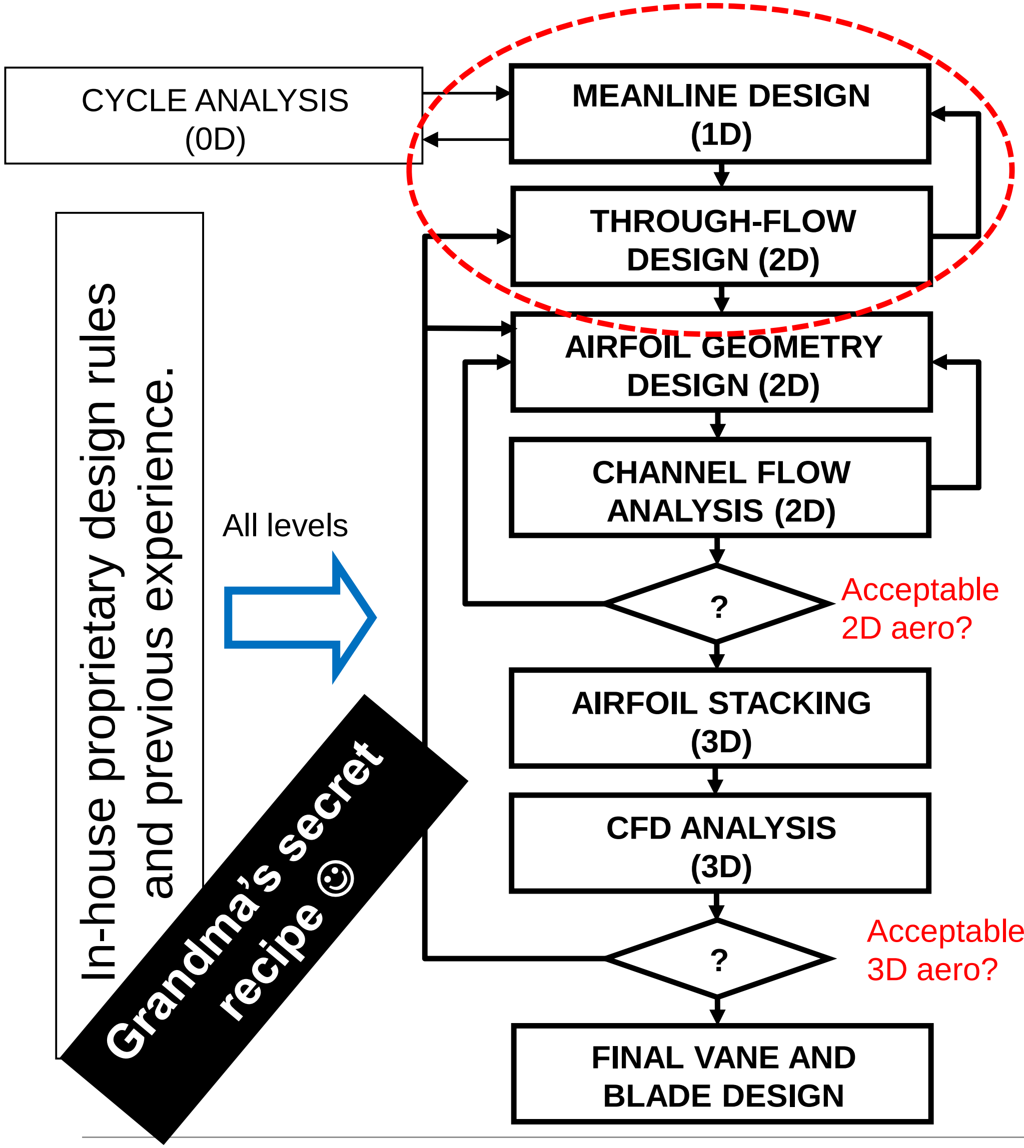
- Improve the accuracy of the 1D turbine design by improved:
  - Exit Mach number correlations
  - Secondary loss correlation for ultra-low aspect ratios
- Reduce the need for “unnecessary” CFD work early in the design process
- Reduce the risk of costly late design changes

The project cont’d

- Background and literature
  - Complete Craig & Cox loss model
- Exit Mach number correlation
  - ~400 tested turbines by CIAM et al
- Low-AR turbine secondary loss correlation
  - CFD
  - 2 LTH “typical” full low-AR stages
- New turbine code
  - Target pressure
  - New improved “physical” angle model
  - Python
  - Real gas
- Outcomes (so far)
  - Report on loss models
  - Report on shock loss model
  - Conference paper is pending



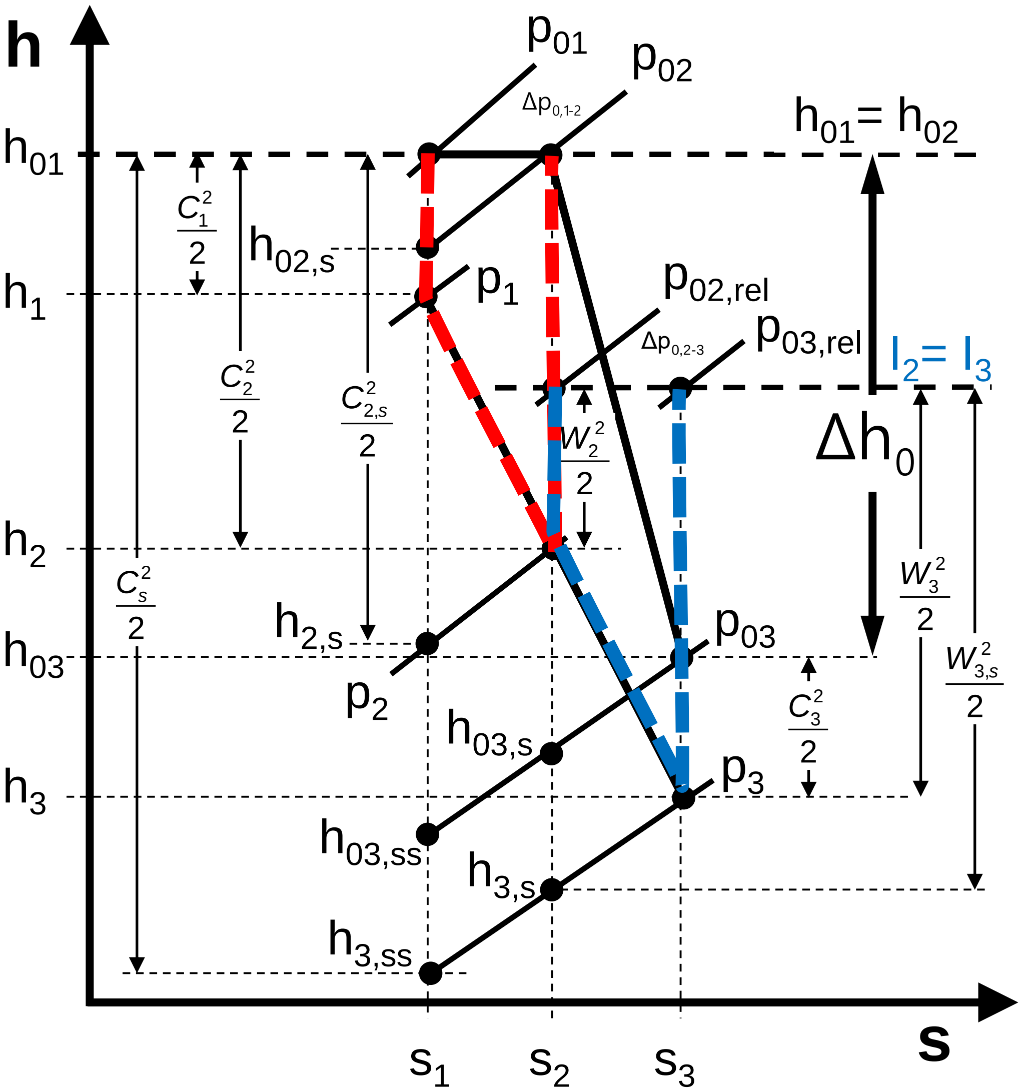
Aero design process



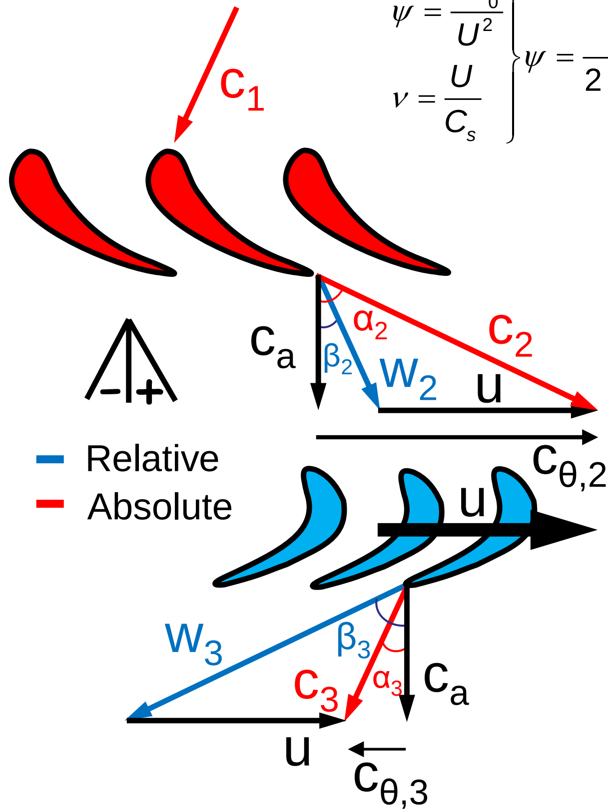
- Reduced order through-flow (free vortex, a=const). Inputs like: angles, loading, reaction, intra stage swirl, diffusion factors, Zweifel number, cooling etc.
- Include hub-to-shroud variation
- Axi-symmetric, non free-vortex
- Design individual airfoil sections (cylindrical)
- Profile family, DCA, CDA, Arbitrary, Bezier, …

Evaluate blade surface Mach numbers and diffusion factors. Determine optimum surface contours by iterating with airfoil geometry program.

Intro to a turbine stage



$$\psi = \frac{\Delta h_0}{U^2}, \quad \nu = \frac{U}{C_s} \; \Bigg\} \; \psi = \frac{\eta}{2 \cdot \nu^2}$$



$$\Delta h_o = U\left(C_{\theta,2} - C_{\theta,3}\right)$$

Ainley and Mathieson et al.

- Mid-span model
  - ✓ A well-established and widely used model (e.g. P&W, Siemens,…)
  - ✓ Normally stated to be within ± one percent
  - ✓ A model is always just a model… Use with caution!
- Several updates since 1951
  - ✓ AMDC+KO+MK+…
- Fairly simple to use – less geometry than e.g. Traupel, Craig & Cox, Denton,…
- Each major component is separately modeled:

$$Y_{TOT} = Y_P \cdot f_{RE} \cdot CFM + Y_S + Y_{TE} + Y_{Clr}$$

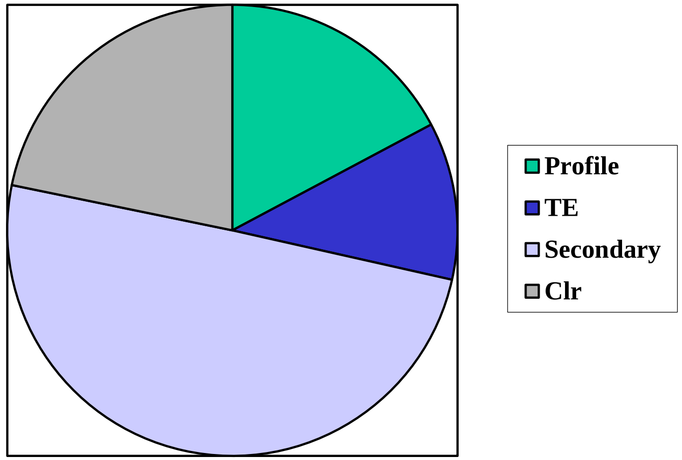
- Y_P: Profile loss
- f_{RE}: Reynolds number correction
- CFM: Shock correction
- Y_S: Secondary loss
- Y_{TE}: Trailing edge loss
- Y_{Clr}: Clearance loss

Super-sonic design example

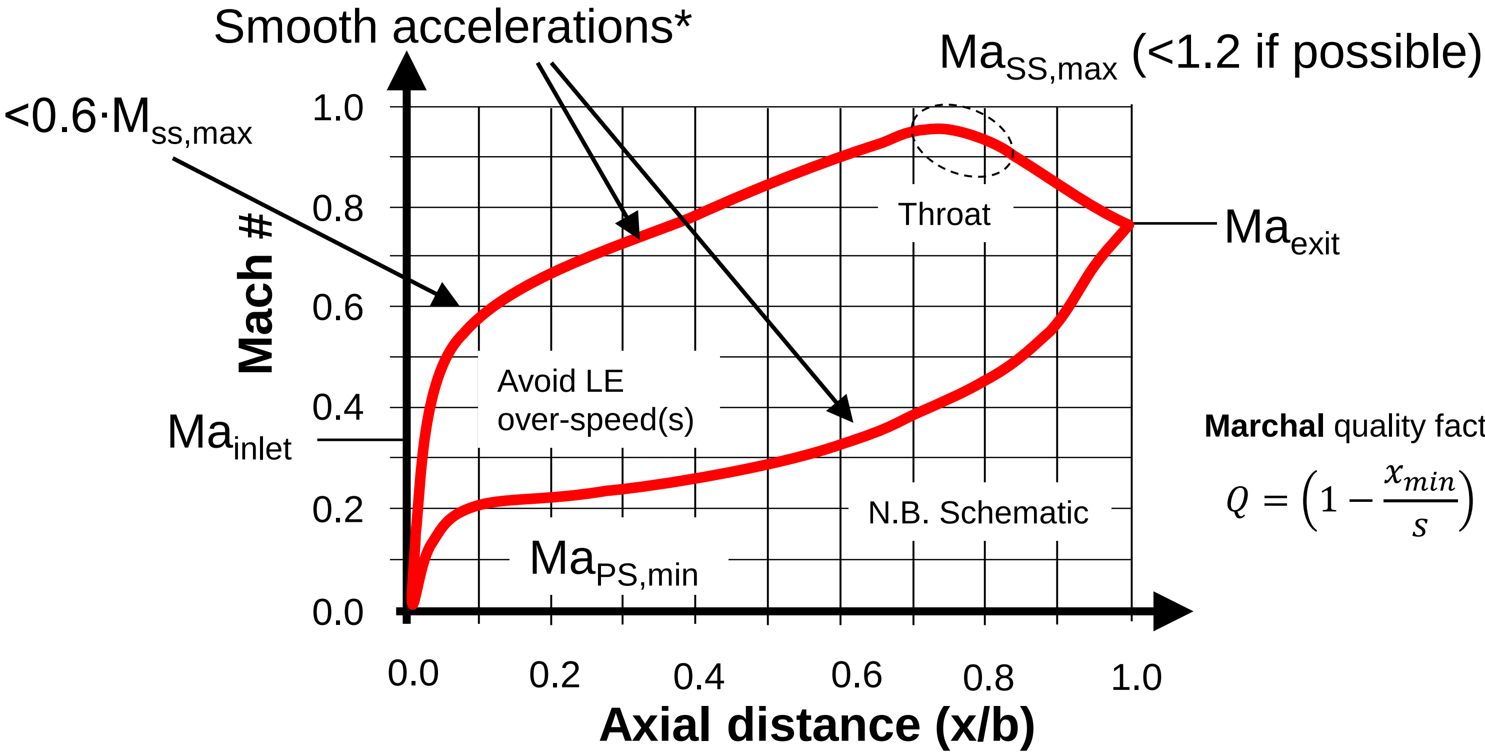
Design point parameters	
Pressure ratio	3.76
Stage loading	2.47
Flow coefficient	0.64
Mean reaction	30%
Mean exit swirl	37°
Efficiency	83.5%

Airfoil parameters	Stator	Rotor
Inlet Mach Number	0.10	0.55
Exit Mach Number	1.10	1.14
Flow turning	76°	124°
Aspect ratio	**0.70**	**1.44**
Zweifel loading	0.84	0.76
Trailing edge/throat	17%	10%
Tip clearance/height	0.0%	1.5%

Aerodynamic losses		Stator	Rotor	Total
Profile	Y_p	0.029 (20%)	0.058 (16%)	0.087 (15%)
Trailig edge	Y_{TE}	0.046 (31%)	0.011 (12%)	0.057 (12%)
Secondary	Y_S	0.071 (49%)	0.180 (51%)	0.251 (50%)
Tip clearance	Y_{clr}	0.0 (0%)	0.110 (30%)	0.110 (21%)
Total	Y	0.147 (100%)	0.356 (100%)	0.500 (100%)



Turbine profiling – loading philosophy



Marchal quality factor:

$$Q = \left(1 - \frac{x_{min}}{s}\right)\left(\frac{b_0}{s}\frac{\Delta p_{max}}{1/2\,\rho c_m^2}\right)$$

$$D_{SS} = \frac{Ma_{SS,max} - Ma_{exit}}{Ma_{SS,max}} \leq 0.20 \ldots 0.25 \qquad D_{PS} = \frac{Ma_{inlet} - Ma_{PS,min}}{Ma_{inlet}} \leq 0.45$$

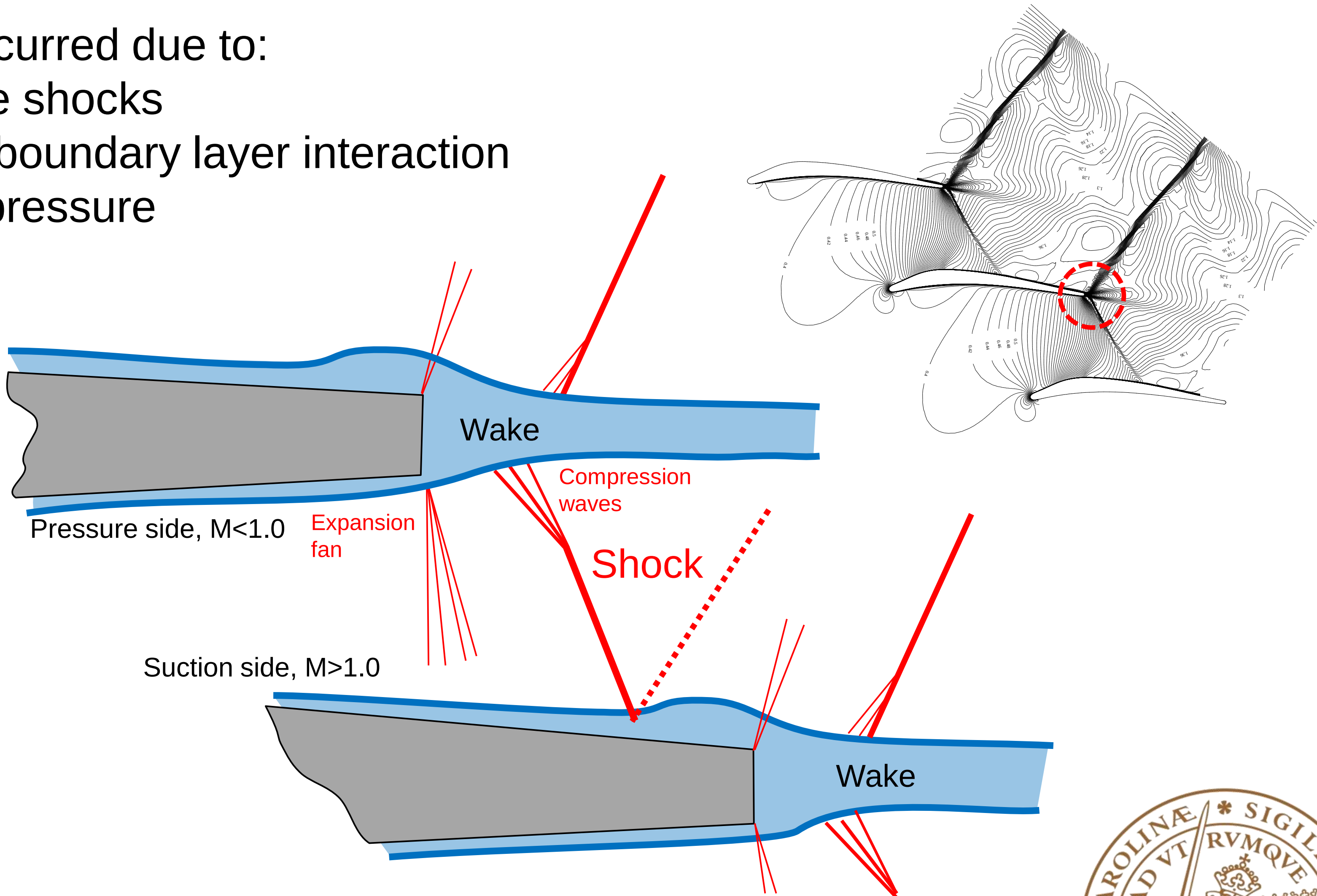
$$Rate\ parameter = \frac{D_{SS}}{\Delta l_{uncov}/l_{SS}} \leq 0.6$$

*Thin boundary layers

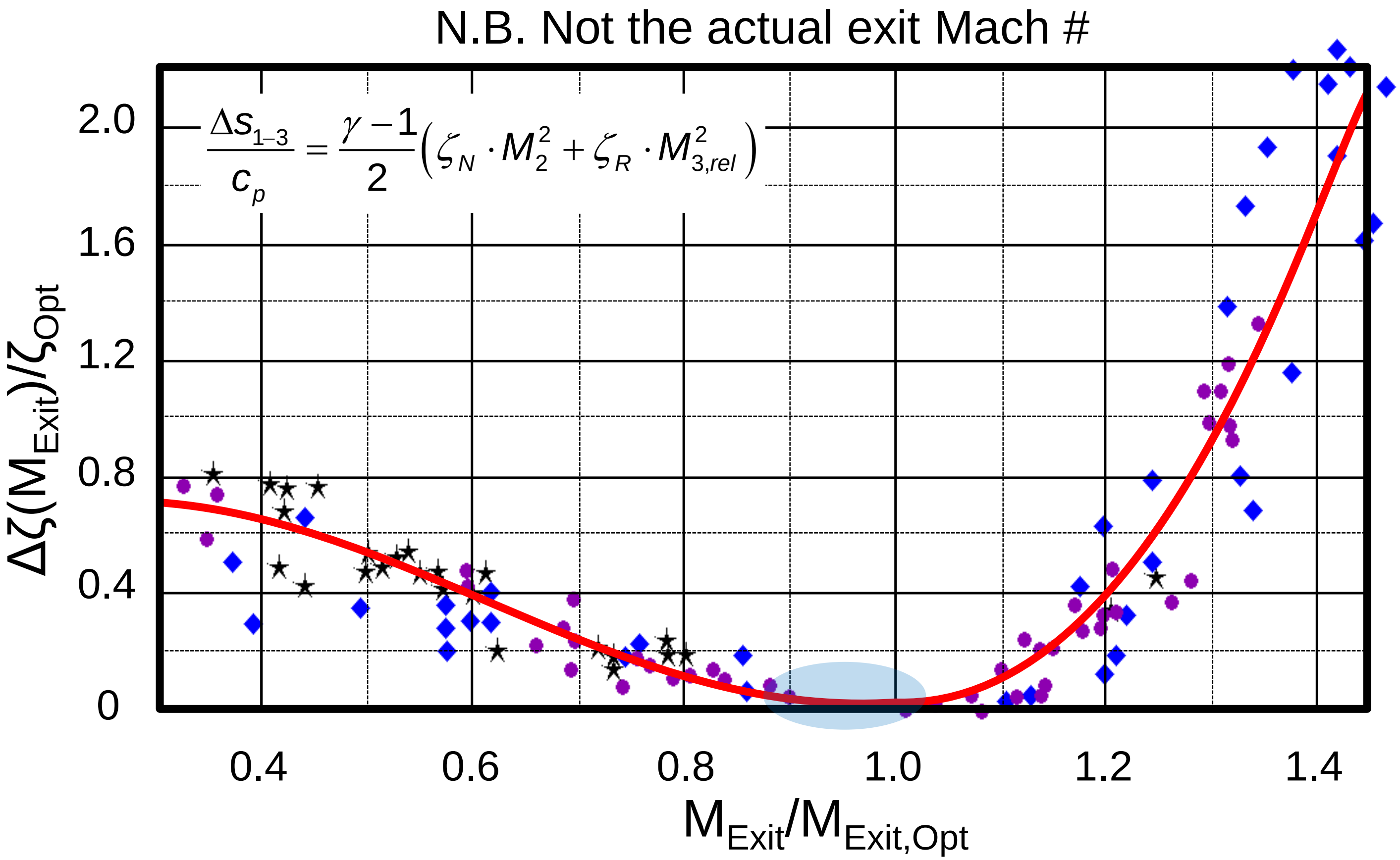
Schematic flow structure

Loss is incurred due to:

- Oblique shocks
- Shock/boundary layer interaction
- Wake pressure



Influence from exit Mach # level (70s)



Based on: Mukhtarov et al. "Procedure of Estimating Flow Section Losses in Axial Flow Turbines when Calculating Their Characteristics, Teploenergetica Vol 16 (7), pp. 76-79

$$\frac{\Delta\zeta(\lambda)}{\zeta_{crit}} = 6.05\left(\frac{\lambda_2}{\lambda_{2,opt}}\right)^3 - 11.8\left(\frac{\lambda_2}{\lambda_{2,opt}}\right)^2 + 6\left(\frac{\lambda_2}{\lambda_{2,opt}}\right) - 0.25$$

Exit Mach # loss correlations

Caveat!
The loss associated with the exit Mach number is, by far, more complicated than indicated by the equations!!!

- The AMDC-KO model for super-sonic exit velocity is:

$$CFM_{KO} = 1 + 60\left(M_2 - 1\right)^2$$

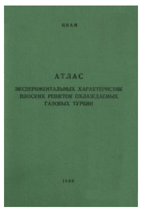
- The previous model is quite conservative and a later correction is by Came:

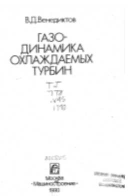
$$CFM_{Came} = 1 + 49.5\left(M_2 - 1\right)^3 + 3.3\left(M_2 - 1\right)^2$$

- Prof. Venediktov (CIAM) presented a, by far, better model in 1990:

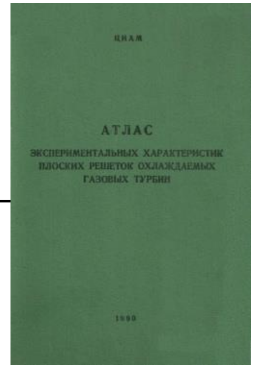
$$\zeta_{Shock}\left(\lambda, \beta_{gauge}, \delta, d_{TE}/o\right) = \left(1.21829 \cdot 10^{-4} \cdot \beta_{gauge}^2 + 1.16320 \cdot 10^{-3} \cdot \delta^2\right) \cdot \omega_\lambda \cdot \frac{d_{TE}}{o}$$

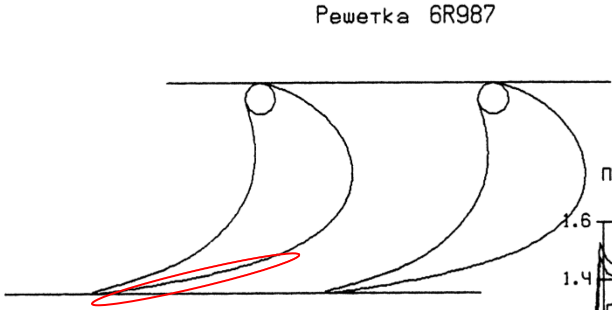
$$\omega_\lambda = \frac{1}{2}\left[\sin\left(2 \cdot \pi \cdot \left\{\lambda - 1.15\right\}\right) + 1\right] \quad 0...1.0 \; when \; \lambda = 0.9...1.3$$

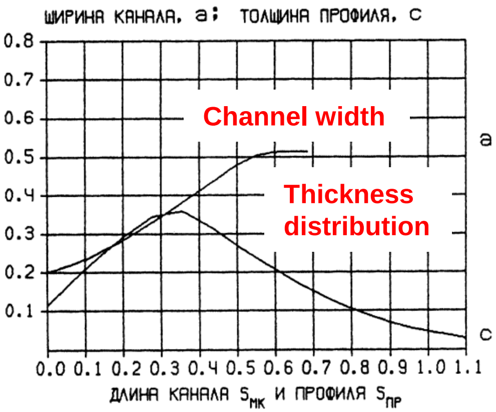


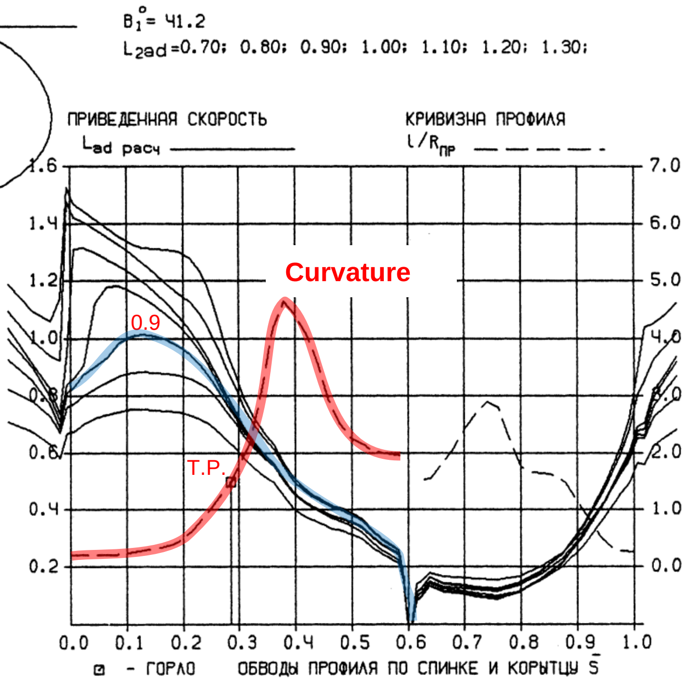


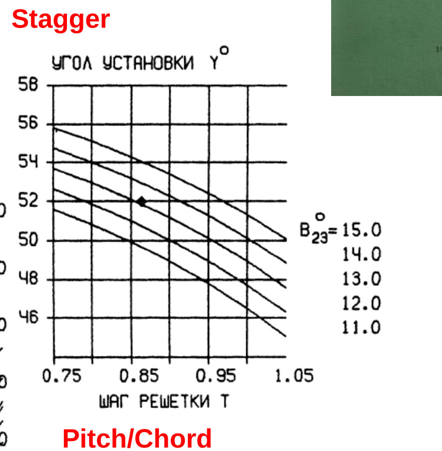
Profile atlas – #53 (of some 400…)

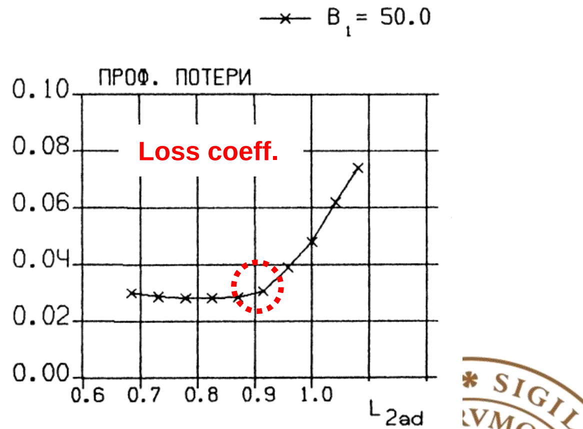












$N_{реш}$	$B_{1К}$°	B_{23}°	$C_М$	T	δ°	d_1	$\bar{d}_2$	γ°	$X_{СМ}$	$Y_{СМ}$	$u_{КР}$°	E_2°	E_1°
53	41.2	13.13	0.368	0.864	21.6	0.115	0.045	52.0	0.187	0.357	-3.01	14.3	0.0

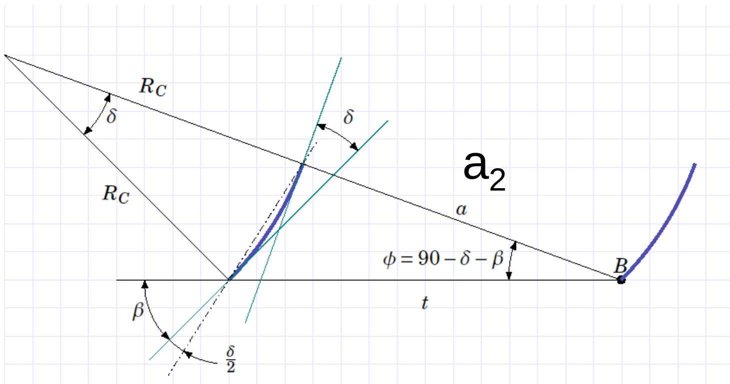
$X_Г$	u_2°	w_1°	w_2°	K	a_1/a_2	$a_1/a_м$	$P_{СП}$	P	F	Ƃ	a_2, мм	l, мм
0.382	6.84	56.34	7.36	2.90	2.62	1.00	1.691	2.787	0.231	0.768	8.86	45.14

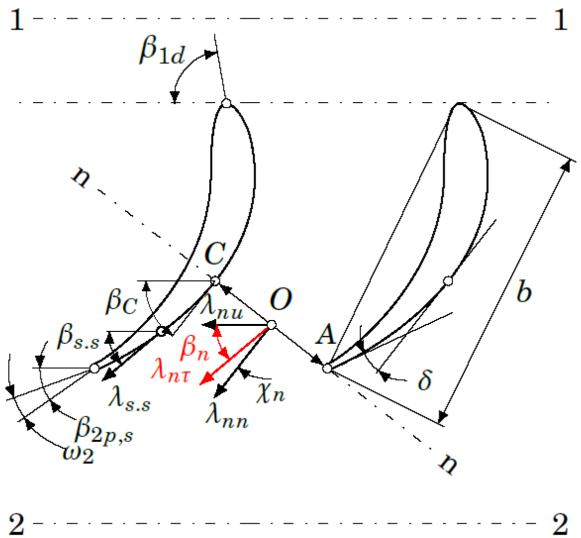
$\beta_{gauge} = \sin^{-1}(o/s)$

$\delta = f\left(Ma_{out}, \beta_{out}\right)$

Surface coordinate (0…1)

Optimum exit Mach (Laval) number





Assume linear dependency with R_c:

$$\lambda_n = \frac{R_C}{R_C + a_2/2} \cdot \lambda_C$$

Conservation of moment at of momentum TE yields:

$$\left.\begin{array}{l} \lambda_2 = \dfrac{\cos \beta_n}{\cos \beta_2 \cdot \left(1 + \dfrac{a_2}{2 \cdot R_C}\right)} \cdot \lambda_C \\ \left.\begin{array}{l} \beta_n = \dfrac{\beta_C + \beta_{2,b}}{2} \\ \beta_C = \beta_{2,b} + \delta/2 \end{array}\right\} \beta_n = \beta_{2,b} + \delta/2 \end{array}\right\} \therefore \lambda_2 = \frac{\cos\left(\beta_{2,b} + \delta/2\right)}{\cos \beta_2 \cdot \left(1 + \dfrac{a_2}{2 \cdot R_C}\right)} \cdot \lambda_C$$

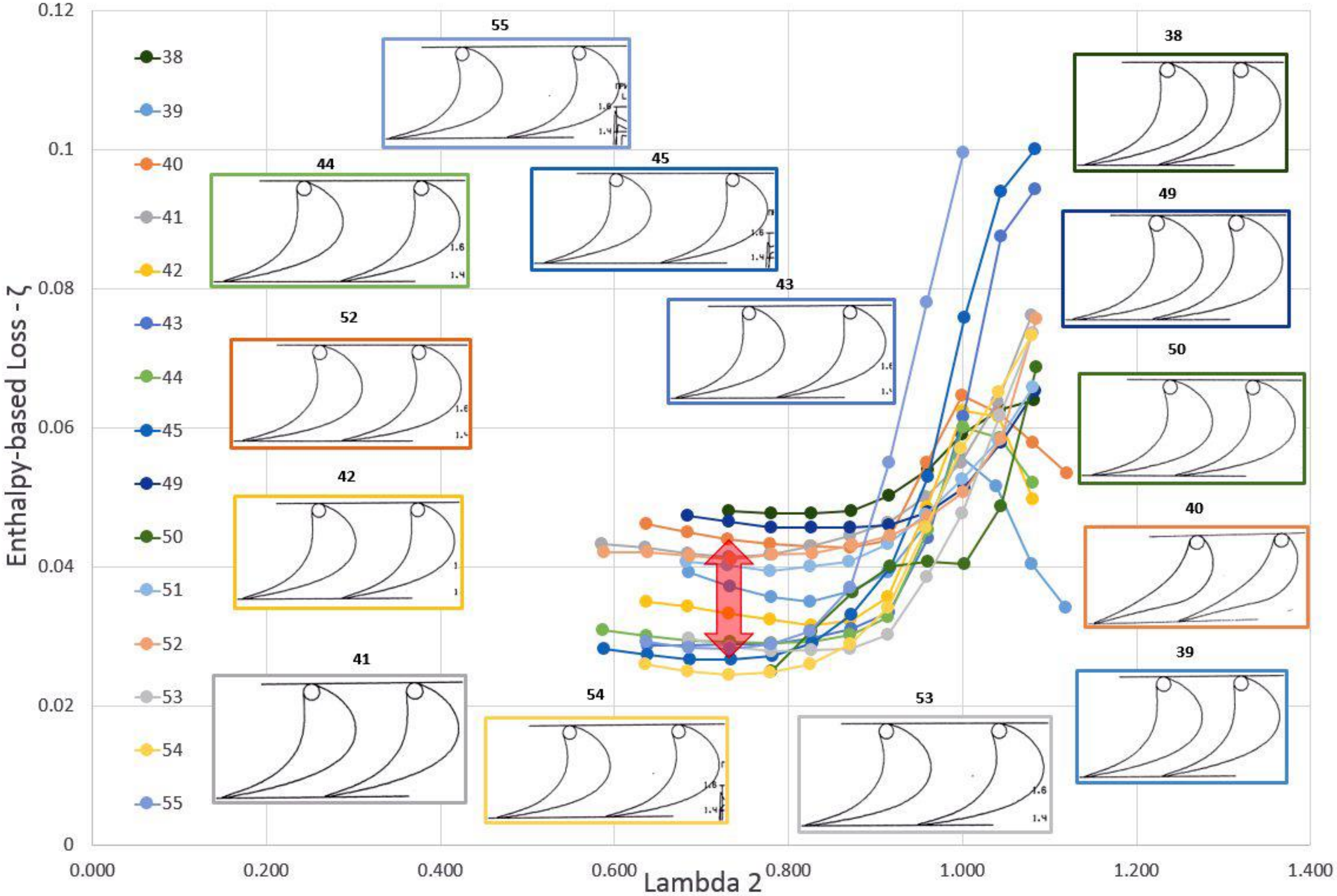
Introduce the expression for the a_2/R_C ratio:

$$\frac{a_2}{R_C} = \frac{2}{\sqrt{1 + \dfrac{\cot^2 \beta_{gauge}}{\sin^2 \delta/2}}}$$

The final relation for isentropic exit velocity yields:

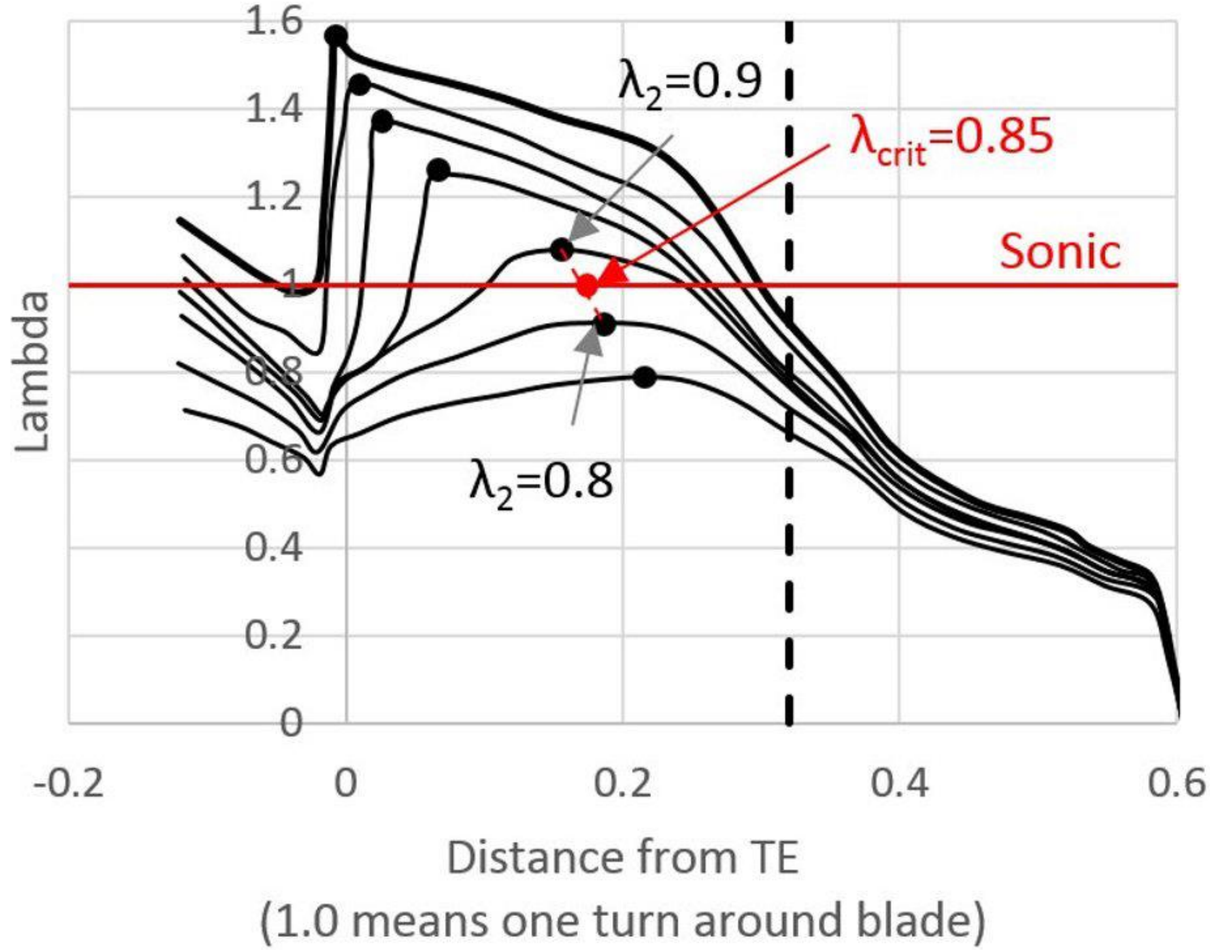
$$\lambda_{2,s} = \frac{\cos\left(\beta_{2,b} + \delta/2\right) \cdot \left(\sqrt{1 + \dfrac{\cot^2 \beta_{gauge}}{\sin^2 \delta/2}} - 1\right)}{\sqrt{1 - \zeta_{th \to TE}} \cdot \cos \beta_2 \cdot \sqrt{1 + \dfrac{\cot^2 \beta_{gauge}}{\sin^2 \delta/2}}} \cdot \lambda_C$$

A collection of blades from the Atlas



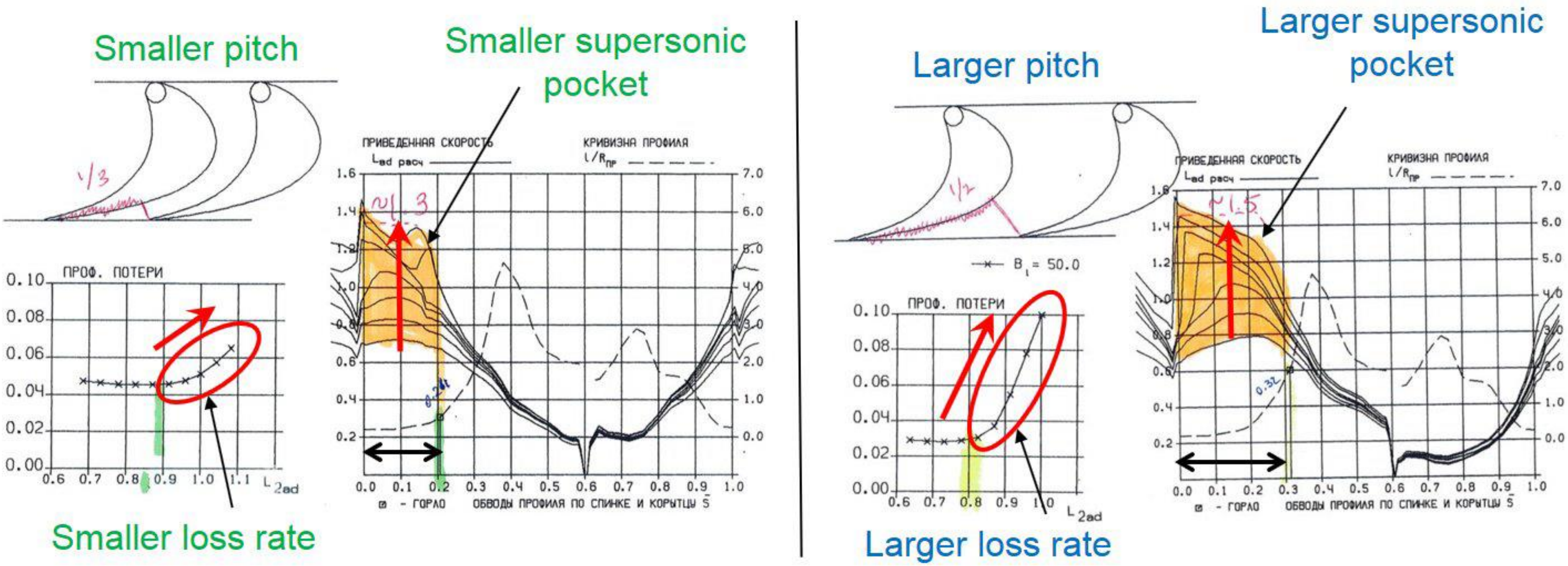
Blade #	t/c	β_2	δ
38	0.555	12.01	5.3
39	0.605	12.53	6.6
40	0.656	13.07	8.2
41	0.706	13.66	10.1
42	0.807	15.00	14.8
43	0.858	15.76	17.6
44	0.908	16.58	20.4
45	1.009	18.37	26.4
49	0.593	9.58	9.3
51	0.729	11.13	14.4
52	0.802	12.13	18.1
53	0.864	13.13	21.6
54	0.927	14.21	25.3
55	1.000	15.57	29.8

Data mining – profile #55



Cascade properties and loss

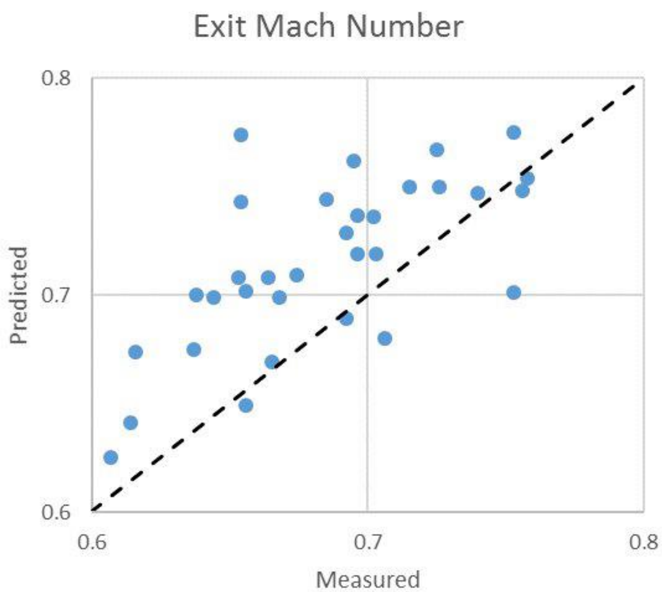
Area ~ lift



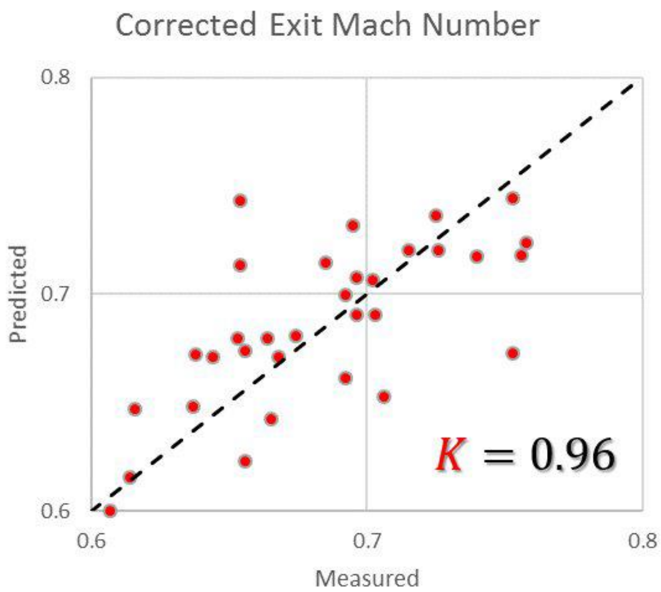
Shock losses should always be added!

Prediction of critical exit Laval

As per Mamaev thesis



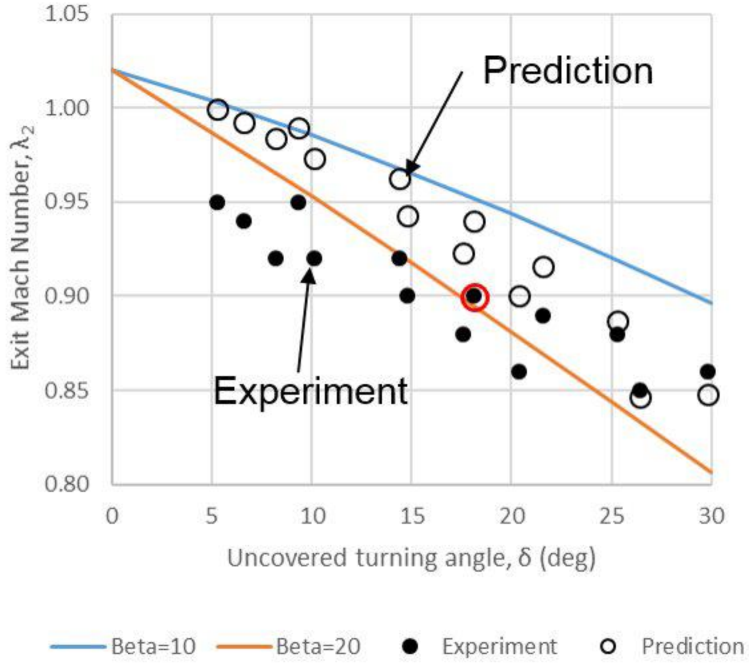
Accounting for Losses with factor K



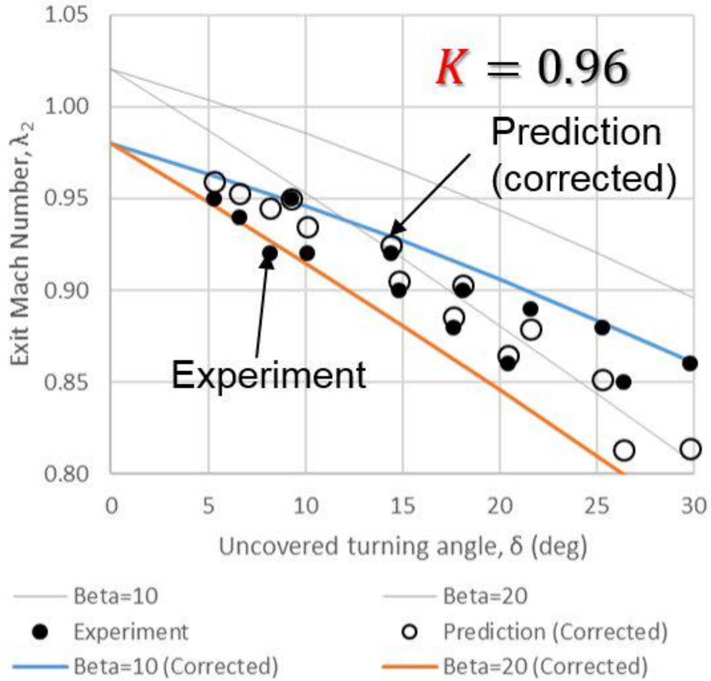
K accounts for losses in angular momentum in the near wall region. In this case, it means an average of 4 % loss of momentum.

Exit Laval # vs. uncovered turning

Mamaev’s approach

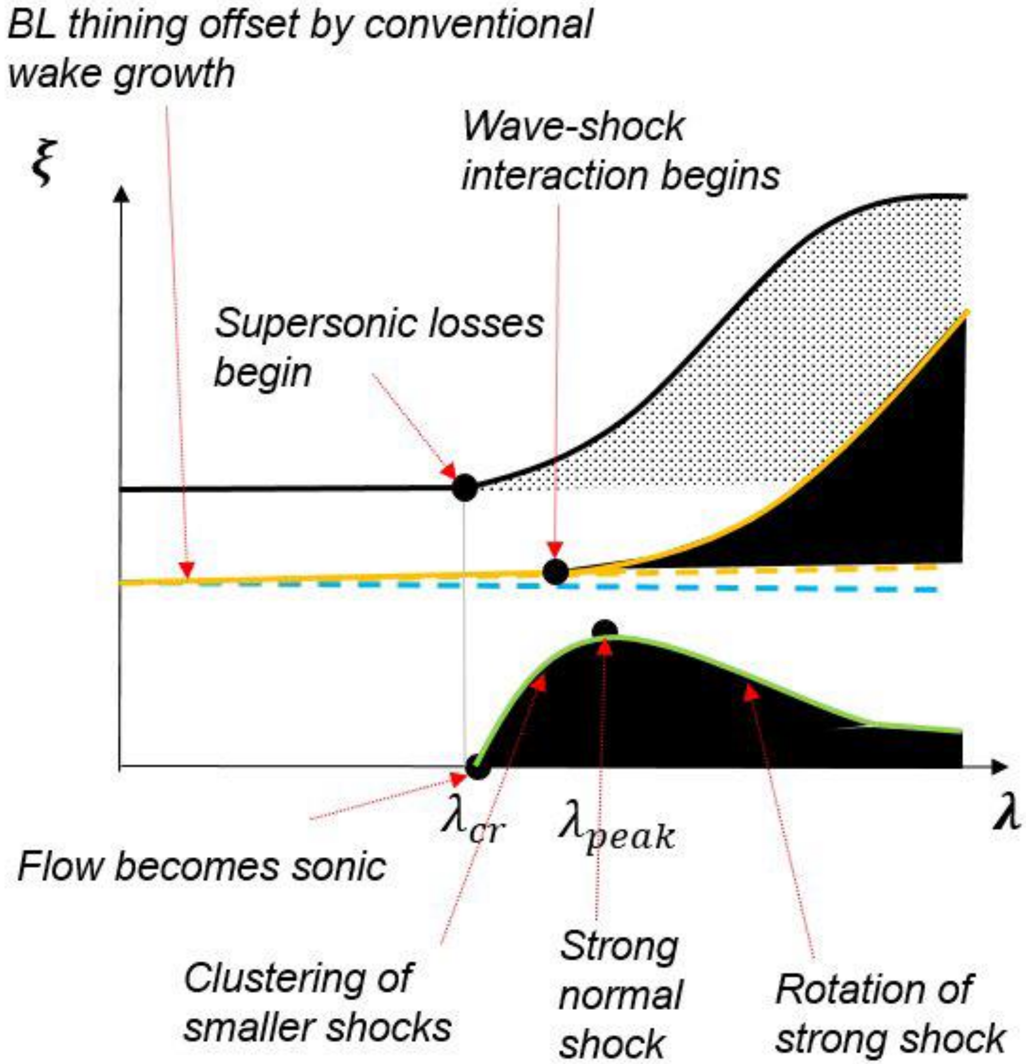


Accounting for Losses with factor K

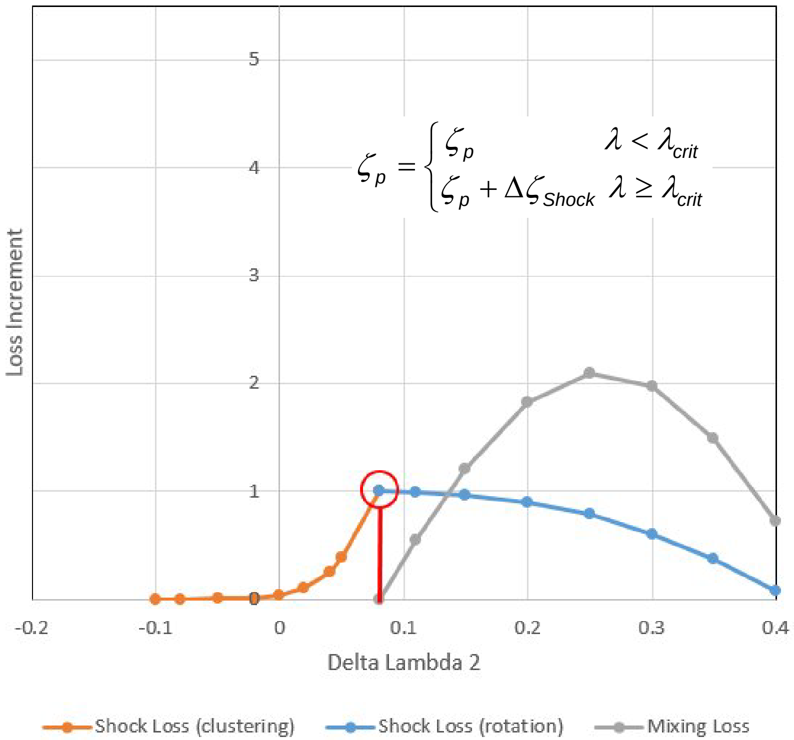


The Teia model #1

$$\zeta_p = \begin{cases} \zeta_p & \lambda < \lambda_{crit} \\ \zeta_p + \Delta\zeta_{Shock} & \lambda \geq \lambda_{crit} \end{cases}$$



The Teia model #2

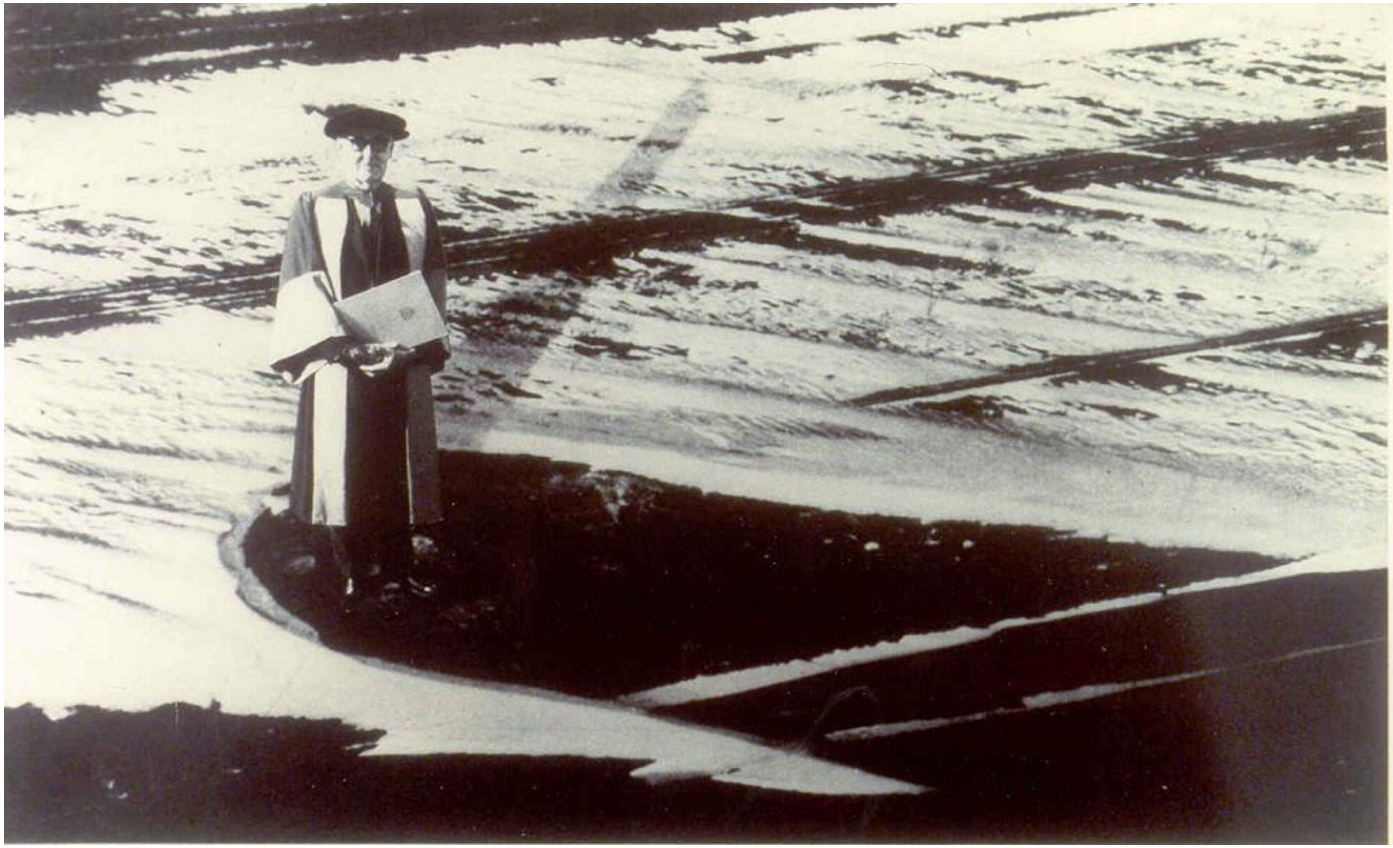


$$\frac{p_{0y}}{p_{0x}} = 1 - \frac{2}{3}\frac{\gamma}{(\gamma+1)^2}\left(M_x^2 - 1\right)^3 + O\left(M_x^2 - 1\right)^3$$

$$\Delta\zeta_{Shock} = \left\{\sin\left(\frac{90}{10^{\left(\lambda_{peak}-\lambda_2\right)/k_1}}\right)\mathrm{K} + \sin\left(\frac{90}{10^{\left(\lambda_2-\lambda_{peak}\right)/k_2}}\right)(\mathrm{K}-1) + k_3\sin\left(\lambda_2 - \lambda_{peak}\right)(\mathrm{K}-1)\right\}\left[1 - \frac{1}{\left(p_{02}/p_{01}\right)^n}\right]$$

$$Where: k_1 = 0.05 \quad k_2 = 1.1 \quad k_3 = 2.1 \quad n = 18$$

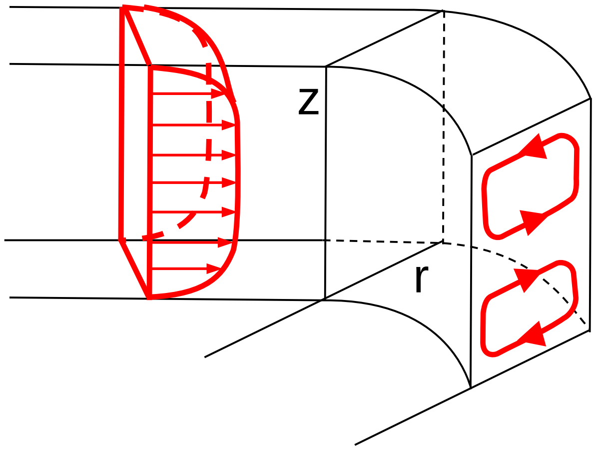
Sir William Hawthorne



Secondary flow in a bend

Two-dimensional velocity distribution

$$\frac{1}{\rho}\frac{dp}{dz} \ll \frac{1}{\rho}\frac{dp}{dr}$$

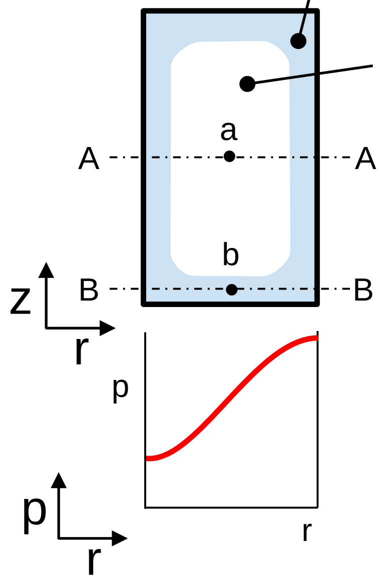


Secondary flow circulation(s)

Generation of secondary flow due to boundary layer entering a curved path

Boundary layer

Flow not effected by viscosity



$$\frac{1}{\rho}\frac{dp}{dz} \ll \frac{1}{\rho}\frac{dp}{dr}$$

$$\frac{1}{\rho}\frac{dp}{dr} = \underbrace{\frac{W_\theta^2}{r}}_{a} > \underbrace{\frac{w_\theta^2}{r}}_{b}$$

Channel X-section and pressure distribution in plane A-A

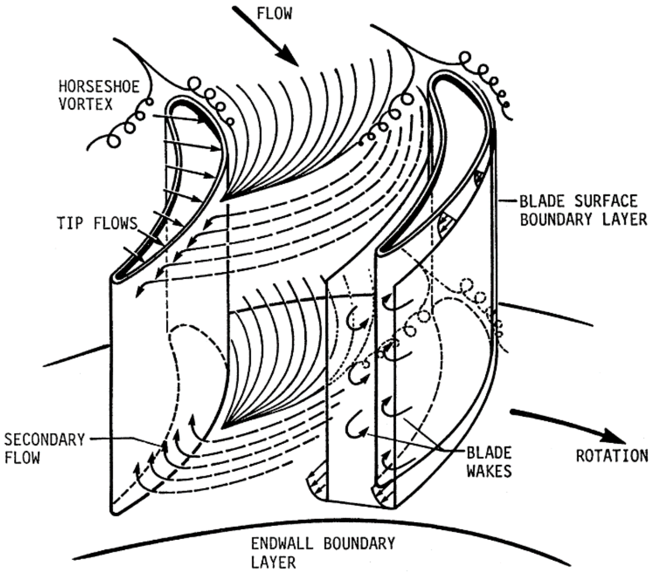
Euler-n equation:

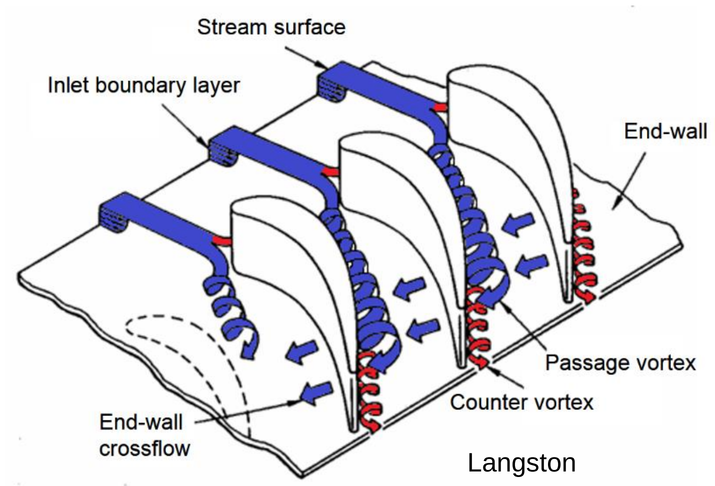
$$\frac{1}{\rho}\frac{dp}{dr} = \frac{W_\theta^2}{r}$$

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{W_\theta^2}{r} = \frac{w_\theta^2}{r} - w_r\frac{\partial w_r}{\partial r} + \text{small viscous terms}$$

Based on Hill & Peterson, “Mechanics and Thermodynamics of Propulsion”

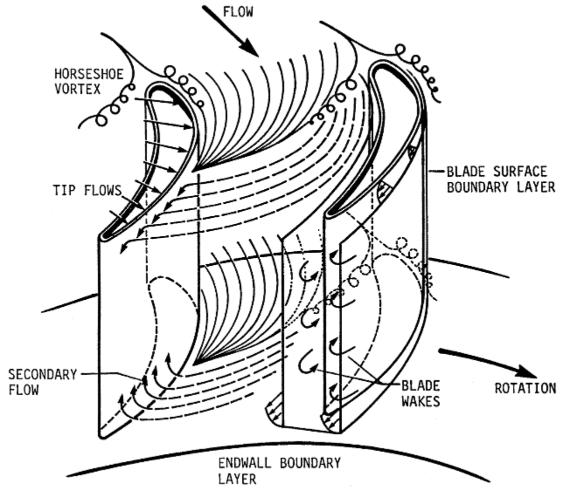
Secondary loss

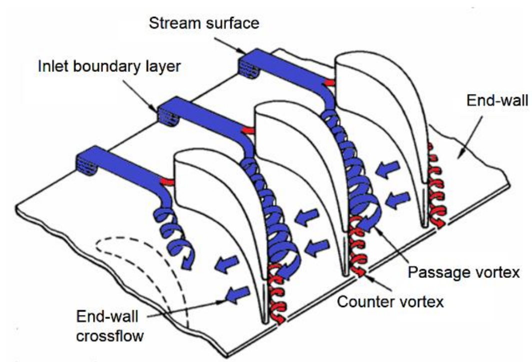




Secondary loss = f(C_L, *Aspect ratio, Turning, Velocity Ratio*)

Secondary loss





Langston

Aspect ratio / Acceleration / Blade lift

$$Y_S^* = 0.0334 f_{AS} \left(\frac{\cos\alpha_2}{\cos\alpha_{1,b}} \right) \underbrace{\left(\frac{C_L}{s/c} \right) \frac{\cos^2\alpha_2}{\cos^3\alpha_m}}_{\text{Ainley loading parameter}}$$

$$\frac{C_L}{s/c} = 2\left(\tan\alpha_1 + \tan\alpha_2\right)\cos\alpha_2$$

$$\alpha_m = \tan^{-1}\left\{ \frac{1}{2}\left(\tan\alpha_1 - \tan\alpha_2\right) \right\}$$

$$f_{AS} = \begin{cases} \dfrac{1-0.25\sqrt{2-h/c}}{h/c} & \text{for } h/c \le 2 \\ \dfrac{1}{h/c} & \text{for } h/c > 2 \end{cases}$$

Doesn't work very well below h/c of unity!

Secondary loss – models

Benner, Sjolander & Moustapha

$$Y_s = \begin{cases} \dfrac{0.038 + 0.41 \cdot \tanh\left(1.20 \cdot \delta^*/h\right)}{\sqrt{\cos(\gamma)} \cdot CR \cdot (h/C)^{0.55} \cdot \left\{\dfrac{C \cdot \cos(\alpha_2)}{C_x}\right\}^{0.55}} & \text{for } h/C \le 2.0 \\[2ex] \dfrac{0.052 + 0.56 \cdot \tanh\left(1.20 \cdot \delta^*/h\right)}{\sqrt{\cos(\gamma)} \cdot CR \cdot (h/C) \cdot \left\{\dfrac{C \cdot \cos(\alpha_2)}{C_x}\right\}^{0.55}} & \text{for } h/C > 2.0 \end{cases}$$

Mamaev et al.

$$\zeta_S = B_S \left(\zeta_P + \frac{2100}{\mathrm{Re}}\right)\frac{a_2}{h}$$

$$B_S = B_\beta \cdot B_h \cdot B_{\mathrm{Re}}$$

$$B_\beta = \underbrace{\left(\frac{19}{3} - \frac{\beta_1 + \beta_{2,gauge}}{30}\right)}_{Turn} \underbrace{\sqrt{\frac{\sin\beta_{2,gauge}}{\sin\beta_1}}}_{Accel}$$

$$B_h = \begin{cases} 1.0 & \text{for } h/a_2 \ge 1.5 \\ 0.66 \cdot h/a_2 + 0.01 & \text{for } 1 \le h/a_2 < 1.5 \end{cases}$$

Denton (CTC)

$$Y_S^* = 0.0334 \cdot f_{AS} \cdot \left(\frac{\cos\alpha_2}{\cos\alpha_{1,b}}\right) \cdot \left(\frac{C_L}{s/c}\right) \cdot \frac{\cos^2\alpha_2}{\cos^3\alpha_m}$$

$$Y_{S,Denton}^* = \frac{3}{8} \cdot Y_S^*$$

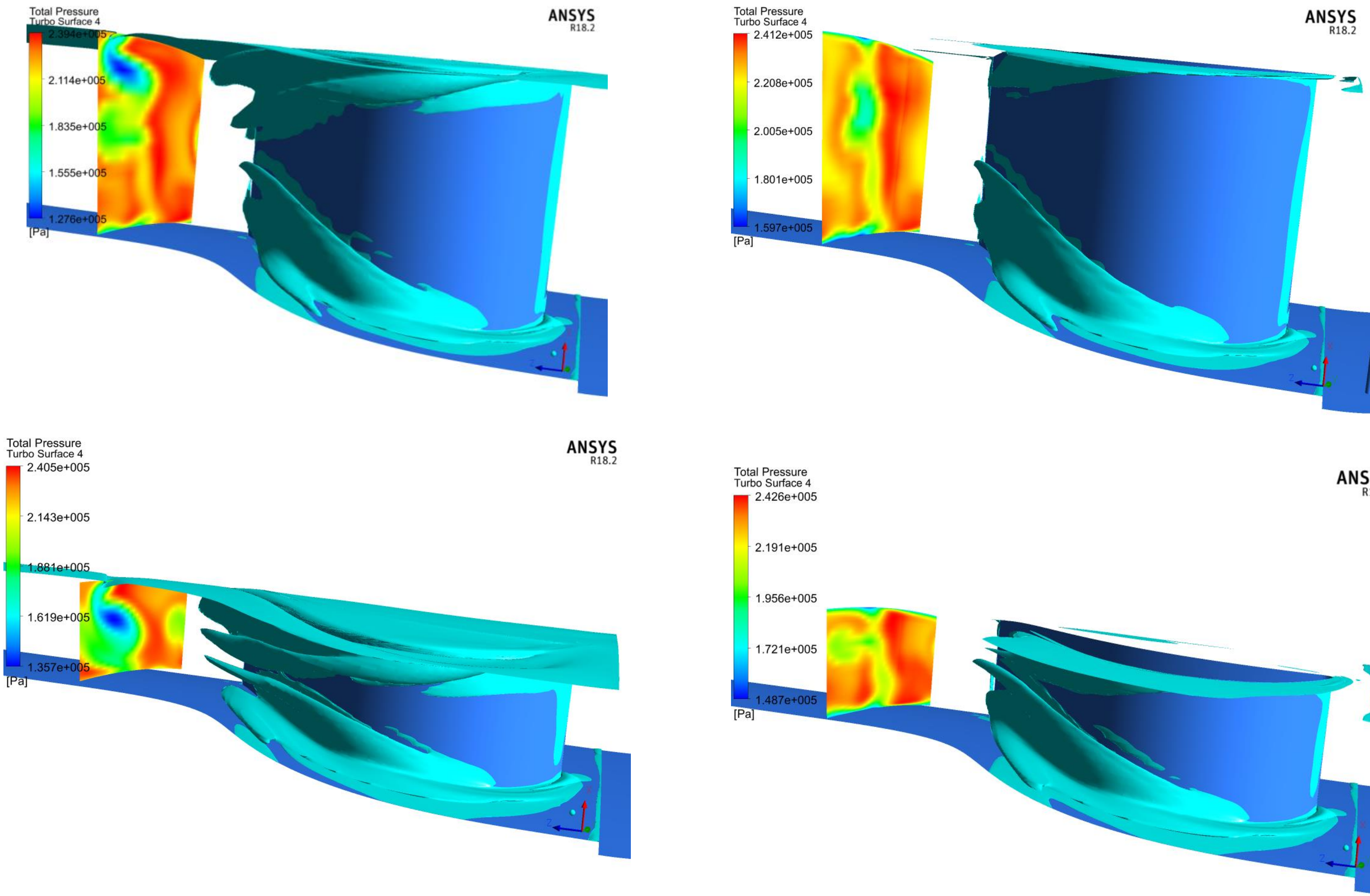
Sharma & Butler

$$\underbrace{\left(1-\varphi^2\right)_{EW}}_{\xi_{EW}} = \underbrace{\left(1-\varphi^2\right)_P}_{\xi_P} \frac{s \cdot \sin(\beta_2) - TET}{h}\left(1 + 4 \cdot \frac{\varepsilon}{\sqrt{CR}}\right)$$

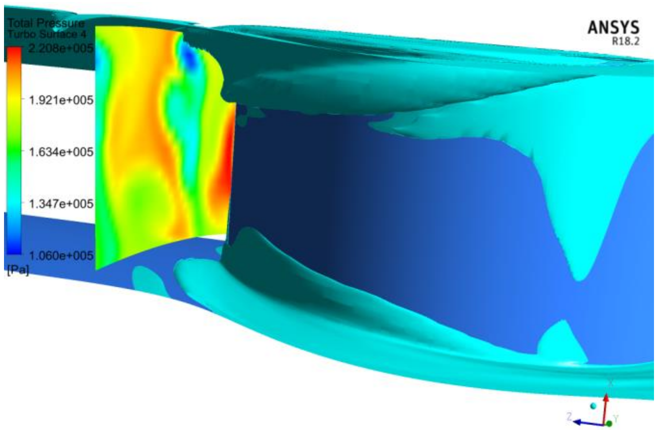
$$\frac{s/c}{AR} = \frac{s/c}{h/c} = \frac{s}{h}$$

Secondary loss = f(C_L, Aspect ratio, Turning, Velocity Ratio)

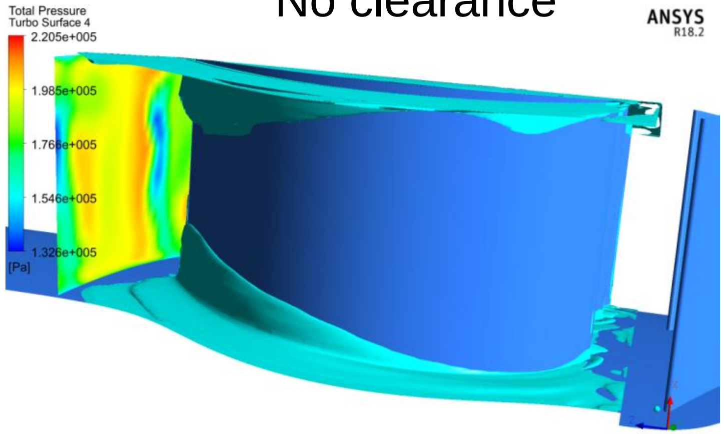
LTH Stages 1 – low and ultra-low AR

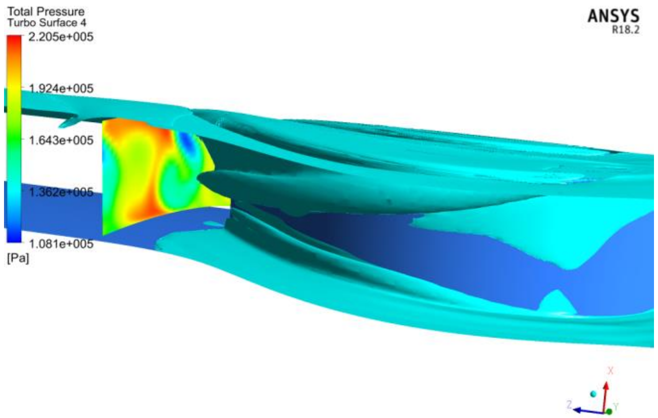


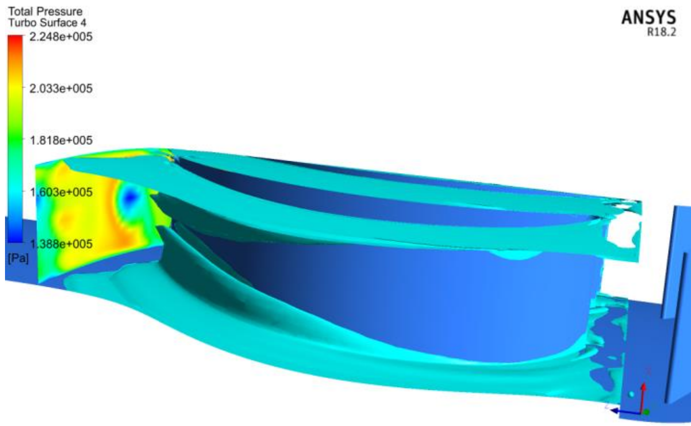
LTH Stages 2 – low and ultra-low AR



No clearance







Thank You!

Questions?